Unclassified

N 66-80624
(ACCESSION NUMBER)
46
(PAGES)
AD-264781
(NASA CR OR TMX OR AD NUMBER)
(THRU)
None
(CODE)
(CATEGORY)
FACILITY FORM 602

Defense Documentation Center

Defense Supply Agency

Cameron Station • Alexandria, Virginia

Unclassified

Agt-32936

A RAND REPORT

PREPARED FOR

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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POWER-BANDWIDTH TRADE-OFFS FOR FEEDBACK FM SYSTEMS: A COMPARISON WITH PULSE-CODE-MODULATION

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RM-2787-NASA

October 1961

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PREFACE

This study was conducted under NASA Contract No. NASr-21 (02), monitored by the Office of Space Flight Programs.

SUMMARY

The discriminator output of an FM receiver can be used to frequency-modulate the local oscillator in such a way as to follow the frequency excursions of the incoming signal and thereby reduce the frequency modulation present at the intermediate frequency. This application of negative feedback has been known for over 20 years, but interest was not stimulated until recently when the principal advantage of feedback FM became apparent, namely its ability to operate at a reduced threshold compared with conventional FM.

The expense and difficulty of placing large weights into earth orbits and interplanetary trajectories has stimulated the exploitation of efficient communication and telemetering techniques which minimize the amount of transmitter power required. Feedback FM appears to be a technique which may find considerable use in these applications.

For a given quality of service, power can be traded against bandwidth depending on the amount of feedback used. Large amounts of feedback in a typical system can reduce the amount of transmitter power required by as much as 8 db while roughly doubling the radio channel occupied, compared with a conventional FM system designed to minimize transmitter power. However, much of the indicated improvement can be attained by the use of relatively small amounts of feedback, i.e., 15 to 20 db. Even if the threshold were to behave in the idealized manner assumed in the analysis, there would be little point in using more than 40 db of feedback.

Assuming an optimum detection process, a comparable PCM system requires about 3 db less power than a conventional toll-quality FM system but about 5 db more power than a feedback FM system using 35 to 40 db of feedback.

However, the PCM system will require only about half the bandwidth of the feedback FM system.

While the computations presented here form an important basis for comparison of PCM and feedback FM systems, the choice in any specific application will also be influenced by considerations of circuit complexity, multiplexing problems, etc.

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LIST OF SYMBOLS

$A(\omega)$	amplitude characteristic of feedback amplifier
B	modulation baseband (cps)
C	signal power at threshold (watts)
D	deviation (cps)
E	pulse energy (joules)
$E(\omega)$	Fourier transform of discriminator output
$e(t)$	discriminator output
F	feedback factor
f_c	carrier frequency (cps)
f_o	local oscillator frequency (cps)
$f(t)$	time variation of instantaneous frequency (sec^{-1})
$g(t)$	output from feedback amplifier
$H(\omega)$	transfer function of feedback amplifier
$h(t)$	impulse response of feedback amplifier
k	pulse shape factor
M	modulation index (deviation ratio)
N	number of PCM digits per pulse code group
n	noise power spectral density (watts/cps)
P	signal power (watts)
p	probability of error
$\frac{S}{N}$	output signal-to-noise (power) ratio
W	signal bandwidth (cps)
$W(\omega)$	output power spectral density (watts/cps)
α	normalized signal energy
β	discriminator slope factor (sec)

$\beta(\omega)$	phase characteristic of feedback amplifier
γ	local-oscillator modulation factor (sec^{-1})

LIST OF FIGURES

I. INTRODUCTION

The application of negative feedback to an FM receiver was described in 1939 by Chaffee[1] and Carson.[2] The technique consisted of frequency-modulating the local oscillator with a portion of the output voltage from the discriminator in such a phase as to reduce the output voltage. Chaffee's concept was to utilize feedback in the receiver to reduce the effective carrier deviation and then to restore the original deviation by increasing the modulation at the transmitter. He compared this system with one using ideal limiters and concluded that there was no significant difference in performance with regard to noise at either high or low levels of disturbance although the use of feedback did result in a reduction of the distortion products produced by receiver nonlinearities.

Subsequently, others pointed out that the principal advantage of the use of feedback resides not in any ability to improve receiver performance above the improvement threshold (where, indeed, no significant change results), but in its ability to reduce the signal level required to achieve the improvement threshold. Morita and Ito[3] in 1960 (with a reference to a 1953 patent), and Felix and Buxton[4] in 1958 reported the application of feedback to the FM receivers employed in microwave radio relay systems. In both cases, threshold improvements were noted, though the performance data presented were somewhat sketchy. The Echo communication satellite experiment of August 1960 was another application of feedback FM which demonstrated its ability to operate at reduced signal levels.

An equivalent system, dating back to 1940, was described by Rodionov[5] in 1960. Called "follow-up tuning," it uses negative feedback to vary the resonant frequency of a narrow-band, intermediate-frequency (I.F.) stage in accordance with the instantaneous frequency of the incoming signal. As with the feedback FM system described above, an improved resistance to the interfering effects of noise is obtained by virtue of the decreased system bandwidth.

The expense and difficulty of placing large weights into earth orbits and interplanetary trajectories has stimulated the exploitation of efficient communication and telemetering techniques which minimize the transmitter power required. Feedback FM appears to be such a technique which may find considerable use in these applications. For instance, Pierce and Kompfner(6) in 1959 proposed a satellite communication system using a modulation index large enough to provide an output signal-to-noise ratio of toll quality while operating at a near-minimum threshold obtained by the use of sufficient negative feedback.

The theoretical and experimental analyses to date have considered only moderate amounts of feedback because of the stability problems which appear, as in any feedback application. Nevertheless, system analyses based on the use of feedback FM frequently contemplate the use of significantly greater amounts of feedback. That such amounts of feedback cannot be attained at present is apparently regarded as a practical, not a theoretical, limitation.

Even if this supposition is true, it would be desirable to examine the implication of the amount of feedback on the other system parameters. This memorandum presents an analysis of the effect of feedback on an idealized FM receiver in which the improvement in the threshold performance is related in a simple manner to the amount of feedback used. It is shown that the amount of feedback can be regarded as the parameter controlling a power-bandwidth trade-off. Finally, comparisons are made with several comparable pulse-code modulation (PCM) systems.

II. THE FEEDBACK FACTOR

An idealized superheterodyne FM receiver using feedback is shown in block diagram form in Fig. 1. The flow of the frequency-modulated signal is indicated by dashed lines, that of the demodulated signal by solid lines. The incoming signal is heterodyned with the local-oscillator output to convert it to the intermediate frequency where it is amplitude-limited and then demodulated to yield the output signal. Negative feedback is achieved by using the output to frequency-modulate the local oscillator in such a way as to follow the frequency excursions of the incoming signal.

Ideal elements and perfect limiting are assumed, so that the FM signal can be characterized completely in terms of its instantaneous frequency alone. Thus, the modulation contained in the incoming signal appears as a time variation $f(t)$ of its instantaneous frequency. Similarly, the local oscillator has a frequency modulation $\gamma g(t)$, when $g(t)$ is the amplified feedback audio signal and γ is the modulation factor of the local oscillator. Assuming the local oscillator frequency to be lower than the carrier frequency,* down-conversion leads to a frequency modulation $f(t) - \gamma g(t)$ at the intermediate frequency. An ideal limiter-discriminator with slope factor β then yields an audio output

$$e(t) = \beta \left[f(t) - \gamma g(t) \right] \tag{1}$$

The feedback amplifier has a transfer function

$$H(\omega) = A(\omega)\, e^{i\beta(\omega)}, \qquad \omega = 2\pi f \tag{2}$$

*The assumption is not essential; the converse assumption leads to the same conclusions.

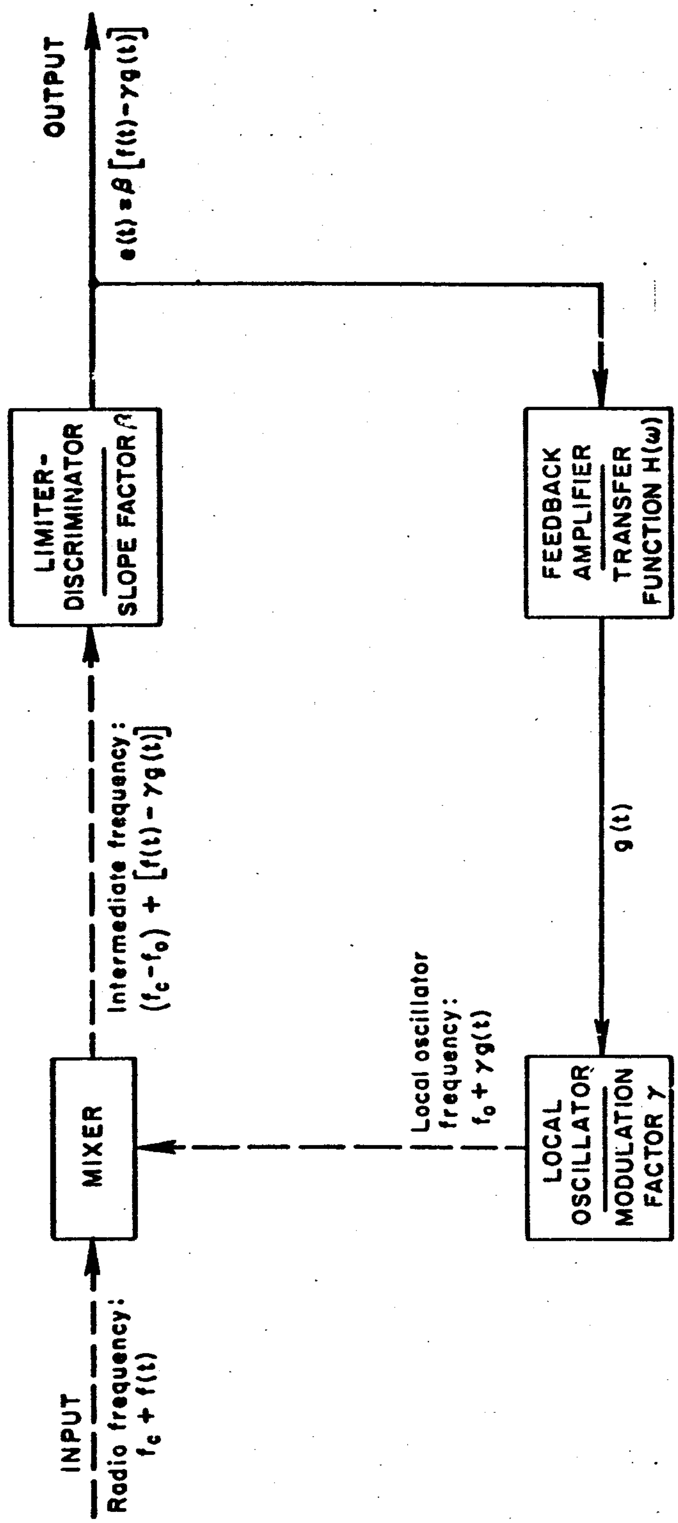

Fig. 1 — Idealized superheterodyne FM receiver with feedback

where $A(\omega)$ is the amplitude and $\beta(\omega)$ the phase characteristic. In terms of the impulse response of the amplifier

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega)\, e^{i\omega t} d\omega \tag{3}$$

the feedback signal can be written

$$g(t) = \int_{0}^{\infty} e(t-T)\, h(T) dT \tag{4}$$

Substituting Eq. (4) in Eq. (1) yields

$$e(t) = \beta \left[f(t) - \gamma \int_{0}^{\infty} e(t-T)\, h(T) dT \right] \tag{5}$$

as the defining relationship between the input $f(t)$ and the output $e(t)$.

Let $e_F(t)$ denote the output with feedback and $e_0(t)$ the output without feedback ($\gamma = 0$). Equation (5) can then be written once for each case and $f(t)$ eliminated between these two equations, yielding

$$e_F(t) + \beta\gamma \int_{0}^{\infty} e_F(t - T)\, h(T) dT = e_0(t) \tag{6}$$

Taking the Fourier transform then gives

$$E_F(\omega) + \beta\gamma E_F(\omega) H(\omega) = E_0(\omega) \tag{7}$$

or, in terms of the output power spectra,

$$W_F(\omega) = \frac{W_0(\omega)}{\left|1 + \beta\gamma H(\omega)\right|^2} \tag{8}$$

The foregoing analysis, though substantially equivalent to that made by Chaffee,[1] has a serious defect which does not permit its free use in predicting the output power spectrum or in determining the stability characteristics of the feedback loop. The deficiency arises from neglecting the effect of the I.F. amplifier on the instantaneous frequency of the signal it passes. This effect can be particularly serious when the passband is made as narrow as possible in an effort to reduce the amount of noise admitted to the limiter-discriminator. Not only is there a change in phase and amplitude of the instantaneous frequency, but distortion terms are also introduced. This makes the system, in reality, a nonlinear feedback network, an exact analysis of which appears extremely difficult.

The principal virtue of the simplified analysis leading to Eq. (8) is that it expresses the outputs in terms of a factor which is particularly meaningful in an FM system. Consider the ratio, F, of the modulation on the incoming signal to that appearing at the intermediate frequency, i.e.,

$$F = \frac{f(t)}{f(t) - \gamma g(t)} = 1 + \beta\gamma \frac{g(t)}{e(t)} \tag{9}$$

where Eq. (1) has been used to eliminate f(t). Now, e(t) and g(t) are the input and output, respectively, of the feedback amplifier so their ratio in Eq. (9), when e(t) is restricted to a single, steady-state sinusoid, is simply the transfer function $H(\omega)$, and the feedback factor can be defined as

$$F = 1 + \beta\gamma H(\omega) \tag{10}$$

For sinusoidal modulation, the feedback factor relates the amplitude and phase of the frequency modulation at the intermediate frequency to that of the incoming signal.

In conventional FM terminology, the magnitude of the feedback factor is therefore equal numerically to the reduction of modulation index (or deviation ratio) which it produces. From a feedback amplifier point of view it also corresponds to the gain of the feedback loop, so the convention of expressing the feedback factor in decibels, i.e.,

$$F = 20 \log \left[1 + \beta\gamma A(\omega)\right] \text{ db} \tag{11}$$

will be adopted for subsequent computational purposes.

III. THRESHOLD EFFECTS

As is well known, a system employing frequency modulation exhibits an improvement-type behavior. That is, the output noise power does not decrease uniformly as the input carrier power is increased. Instead, a quieting effect occurs at a level of carrier power referred to as the improvement threshold. In the vicinity of this improvement threshold, the output noise power varies abruptly. Typical quieting curves are shown in Fig. 2 to illustrate this behavior.* For satisfactory performance, the system must operate above its improvement threshold.

Above the improvement threshold, the output signal-to-noise ratio of an FM system using a modulation index M for a modulation baseband B is given by[8]

$$\frac{S}{N} = 3M^2 \frac{P}{2Bn} , \qquad M = \frac{D}{B} \tag{12}$$

where P/n is the ratio of carrier power to the noise power spectral density** and D is the carrier deviation. The factor $3M^2$ is referred to as the FM improvement factor as compared to amplitude modulation.

A useful approximation to the bandwidth W occupied by an FM signal is given by[6]

$$W = 2(1 + M)B \tag{13}$$

---

*Figures 2 and 3 are adapted from Stumpers (Ref. 7, Figs. 2 and 3).

**Since these quantities appear in ratio, they may be considered at any convenient point in the receiver (prior to the limiters) beyond which no significant noise contributions are made. A point early in the I.F. amplifier is convenient for this purpose.

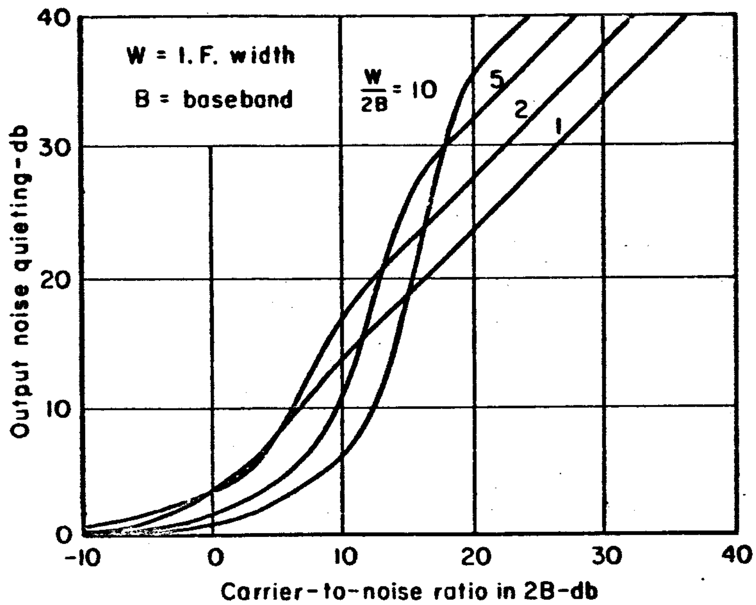

Fig. 2 — Typical FM quieting curves

and a filter with a passband given by Eq. (13) is normally considered to be the narrowest through which an FM signal can be transmitted with tolerable (in terms of voice quality) distortion. The I.F. amplifier of a communication receiver is usually designed to have a near-flat transmission characteristic, with a bandwidth given by Eq. (13), in order to minimize the noise power appearing at the discriminator input (thereby minimizing the carrier power required to achieve the improvement threshold).

When feedback is employed in the receiver, the modulation index, and consequently the signal bandwidth at the intermediate frequency, is reduced. If the modulation index at the radio frequency is M, then the modulation index at the intermediate frequency becomes M/F and the spectral widths of the FM signals at the two frequencies can be written

$$W_{RF} = 2(1 + M)B \; , \qquad W_{IF} = 2(1 + M/F)B \tag{14}$$

respectively.

If there were no other considerations, the bandwidth of the I.F. amplifier could simply be narrowed to the reduced amount given by Eq. (14). However, stability criteria as presently understood appear to prohibit such an approach. Consequently, as the I.F. amplifier characteristic is forced to depart from the rectangular, it can be expected that both the shape of the improvement-threshold characteristic and the carrier-to-noise ratio at which it occurs will be affected.

Further analysis is certainly necessary before these questions can be resolved, but it seems reasonable to speculate that the simplified approach of narrowing the I.F. bandwidth will yield results which represent a theoretical limit to the improvement which can be obtained by the use of feedback. The system analysis to follow is based on such an assumption.

For conventional FM, the improvement threshold is found to occur when the carrier-to-noise ratio at the intermediate frequency is about 12 db,*(6) so the carrier-to-noise ratio at the improvement threshold will be taken as

$$\frac{P}{2(1 + M/F)Bn} = 16 \tag{15}$$

Practical systems using small or moderate amounts of feedback will probably come close to achieving the threshold condition assumed. Systems with large amounts of feedback, as presently conceived, can be expected to fall short of this ideal. Fortunately, the effect of this failing is mitigated by the fact that the analysis presented in the next section indicates that large amounts of feedback are not necessarily desirable, even on a purely theoretical basis; first, because only a small actual carrier power saving remains beyond that yielded by moderate amounts of feedback and, second, because the system bandwidth required when using large amounts of feedback may prove excessive.

The threshold characteristics are illustrated in Fig. 3 for a conventional FM system having a modulation index of 9. According to Eq. (14), the signal bandwidth at both the radio and intermediate frequencies is 20B so, from Eq. (15), the carrier-to-noise ratio at the improvement threshold (referred to a bandwidth 2B) becomes 22 db. The output signal-

*There is an apparent inconsistency of about 2 db between this value and the 10-db threshold which might be inferred from Figs. 2 and 3. The choice of the level of carrier-to-noise ratio at which the improvement threshold occurs is a subjective matter at best since a drastic change takes place in the character of the output noise in its vicinity. This, plus the fact that the 10-db figure derives from idealized theoretical considerations, probably accounts for the difference.

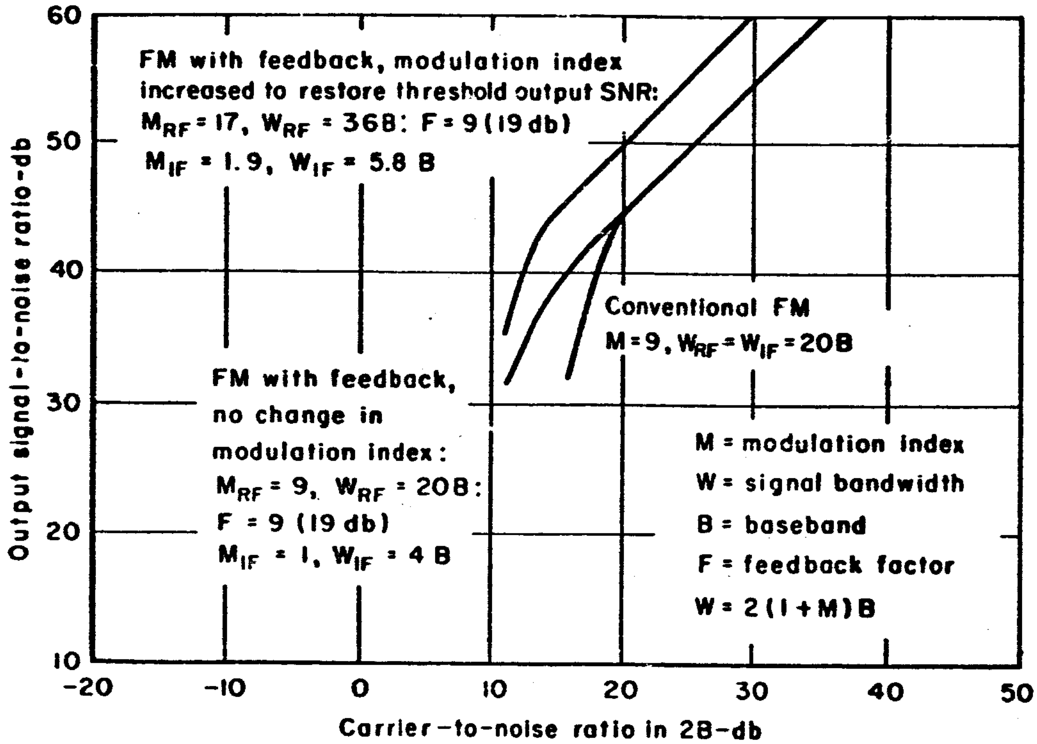

Fig. 3 — Output and threshold characteristics for feedback FM systems

to-noise ratio at the improvement threshold is given by Eq. (12) as 46 db. Below the improvement threshold the output signal-to-noise ratio falls off rapidly; above, it increases linearly with the carrier level.

Applying a feedback factor of 9 (19 db) at the receiver, for example, reduces the modulation index at the intermediate frequency to unity and the signal bandwidth to 4B. This reduction permits the I.F. bandwidth to be decreased by a factor of 5 with a resulting noise reduction of 7 db. Consequently, the threshold carrier power can also be reduced by the same amount.

Unfortunately, the output signal-to-noise ratio at the new improvement threshold is also reduced by the 7-db factor. For some applications this could be viewed as an increased margin against dropout, but it is only obtained by providing degraded service below the original improvement threshold. If a minimum output signal-to-noise ratio is stipulated for a system and the system is designed to provide this service at the improvement threshold, then the use of feedback at the receiver alone is of no real value. From the system point of view, it is necessary to increase the modulation index and thereby restore the original threshold output signal-to-noise ratio.

For the example chosen, the modulation index should be increased to 17.* The feedback reduces this to 1.9 at the intermediate frequency, with a resulting signal bandwidth of 5.8B. Compared to the original bandwidth of 20B, a carrier power saving of 5.4 db becomes possible. The increased FM improvement permits the system to achieve the output signal-to-noise ratio of 46 db at threshold despite the reduced threshold carrier-to-noise ratio.

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*As will be shown later, see Fig. 5.

Raising the modulation index to 17 increases the bandwidth of the radiated signal to 36B in contrast with the original value of 20B. This bandwidth increase by a factor of 1.8 is the trade-off required to realize the carrier power reduction of 5.4 db. In general, the extent and nature of this trade-off between power and bandwidth will depend on the output signal-to-noise ratio desired at the improvement threshold and on the amount of feedback used in the receiver.

IV. SYSTEM PERFORMANCE

For a specified minimum output signal-to-noise ratio and a given amount of feedback, the simultaneous solution of Eqs. (12) and (15) will yield a unique, optimum modulation index. The result of using other values is illustrated in Fig. 4. At lower values of modulation index the improvement threshold is lowered but so also is the threshold output signal-to-noise ratio. Consequently, the carrier power must be increased to yield the desired output quality. From Eq. (12), the relationship becomes

$$\frac{P}{P_{opt}} = \left(\frac{M_{opt}}{M}\right)^2 , \qquad M < M_{opt} \tag{16}$$

The result applies whether or not feedback is used, and is seen to be substantial even for relatively modest decreases.

The use of a modulation index larger than the optimum also increases the power required. In this case, however, the increase is needed simply to achieve the improvement threshold, though, once attained, the output signal-to-noise ratio becomes greater than the minimum specified. From Eq. (15) it follows that

$$\frac{P}{P_{opt}} = \frac{F + M}{F + M_{opt}} , \qquad M > M_{opt} \tag{17}$$

Though extravagant in the use of spectrum, the penalty is not so severe; for systems with feedback it is further reduced.

To differentiate between general conditions and those at threshold, let C denote the level of carrier power at which the improvement threshold occurs. From Eq. (15) the threshold carrier-to-noise ratio becomes

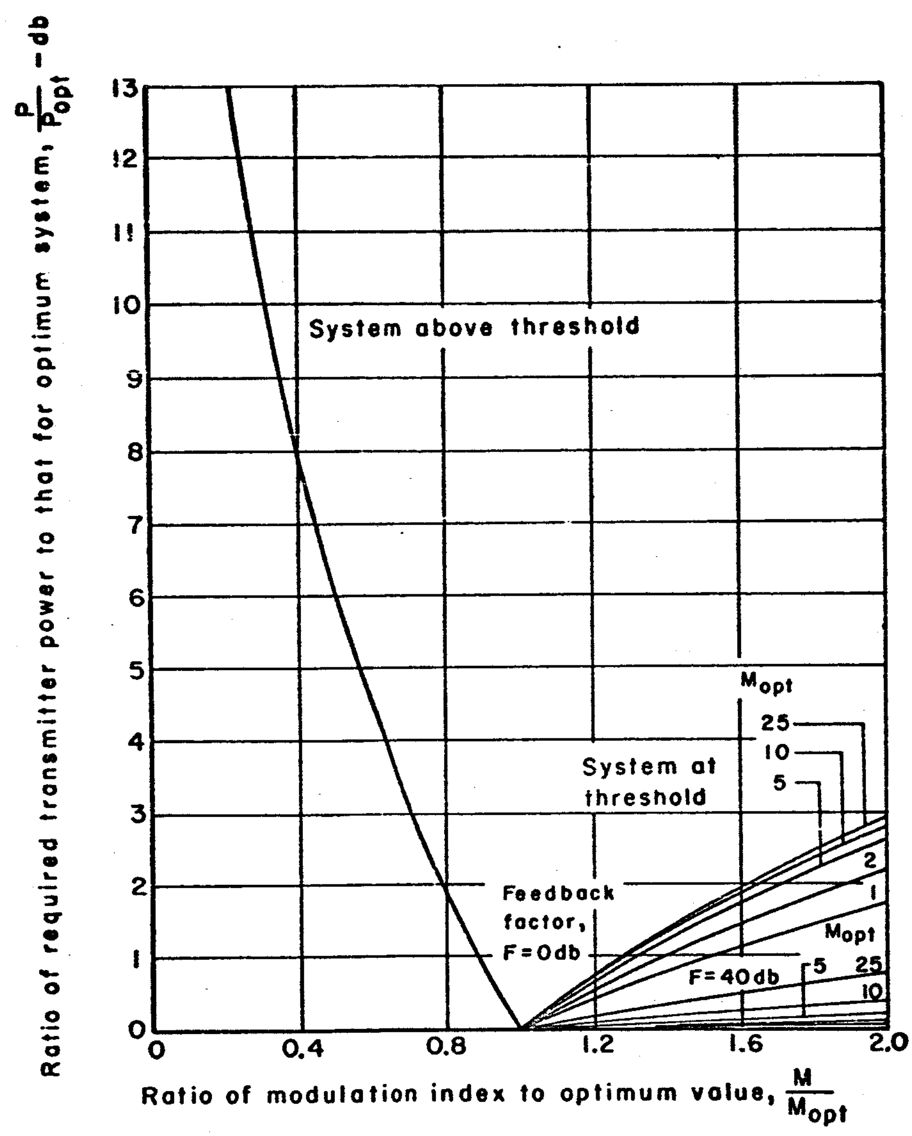

Fig. 4 — Power penalties for non-optimum system design

$$\frac{C}{2Bn} = 16\left(1 + \frac{M}{F}\right) \tag{18}$$

Substituting in Eq. (12) then yields

$$\left(\frac{S}{N}\right)_{\text{thresh}} = 48\, M^2\left(1 + \frac{M}{F}\right) \tag{19}$$

for the threshold output signal-to-noise ratio. A plot of Eq. (19) is given in Fig. 5 for threshold output signal-to-noise ratios of from 20 to 50 db for various amounts of feedback. In accordance with Eq. (11), 0 db indicates a feedback factor of unity (i.e., no feedback, as in a conventional FM receiver*), 20 db indicates a feedback factor, and hence, a modulation-index reduction, of 10, etc.

The power-reduction ratio for FM systems using feedback can be obtained from Eq. (12) by writing it at threshold for systems with and without feedback, then equating these to express corresponding threshold performance. This yields

$$\frac{C_F}{C_0} = \left(\frac{M_0}{M_F}\right)^2 \tag{20}$$

where the subscript denotes the feedback in decibels. The bandwidth-increase ratio is obtained directly from Eq. (14) as

$$\frac{W_F}{W_0} = \frac{1 + M_F}{1 + M_0} \tag{21}$$

*It is interesting to note that the modulation index of 5 employed in commercial FM broadcasting assures an output signal-to-noise ratio of 39 db at threshold. High-fidelity reproduction of music is presumably not possible at lower signal-to-noise ratios, hence, the addition of feedback to a receiver alone would not be profitable; however, the reduction of intermodulation products[1] may prove significant.

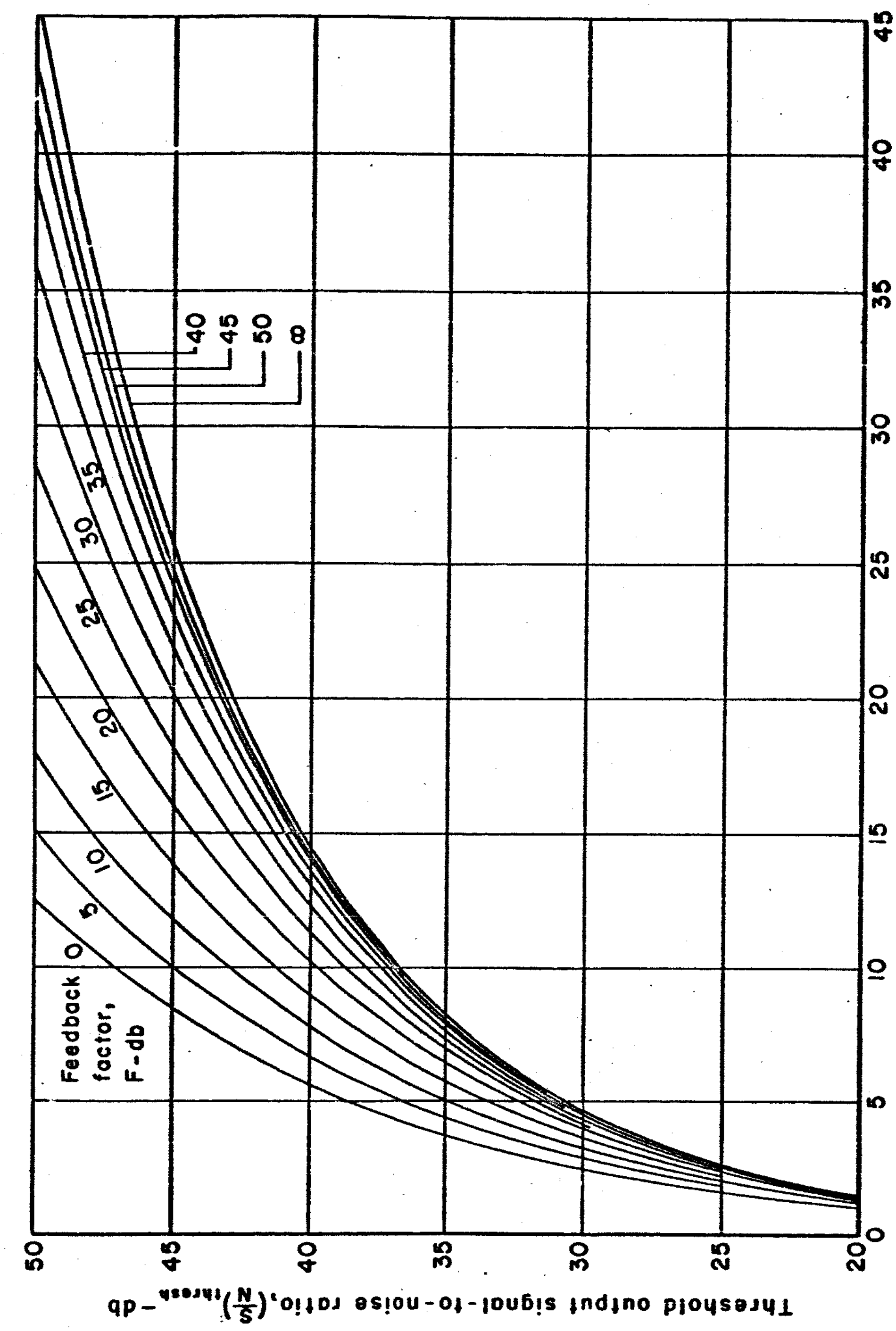

Fig. 5 — Design parameters for optimum feedback FM systems

The values of M_0 and M_F to be used in Eqs. (20) and (21) are the solutions to Eq. (19) given in Fig. 5. These ratios are plotted in Fig. 6 as a power-bandwidth trade-off.

The general behavior at the extremes of threshold output signal-to-noise ratios can be deduced by substituting Eq. (21) into Eq. (20), giving

$$\frac{C_F}{C_0} = \frac{1}{\left[\frac{1}{M_0}\left(\frac{W_F}{W_0} - 1\right) + \frac{W_F}{W_0}\right]^2} \tag{22}$$

and noting that

$$\frac{C_F}{C_0} \approx \left(\frac{W_0}{W_F}\right)^2 \quad \text{for} \left(\frac{S}{N}\right)_{\text{thresh}} \text{large}^{*} , \tag{23}$$

and**

$$\frac{C_F}{C_0} \approx \left(\frac{W_0}{W_F}\right)^4 \quad \text{for} \left(\frac{S}{N}\right)_{\text{thresh}} \text{small}^{*} , \tag{24}$$

The inverse second- to fourth-power trade-offs of carrier power and channel bandwidth are readily observed in Fig. 6, but the effect of feedback is obscured. The same trade-off is shown as an isometric relief in Fig. 7,

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*See Figure 5:

$M_0 \gg 1$ when $\left(\frac{S}{N}\right)_{\text{thresh}} \geq 50$ db, $M_0 \approx 1$ when $\left(\frac{S}{N}\right)_{\text{thresh}} = 20$ db.

**Let $W_F/W_0 = 1 + \Delta$ in Eq. (22). Then, for $M_0 \approx 1$ and Δ small,

$$\frac{C_F}{C_0} \approx \frac{1}{(1 + 2\Delta)^2} \approx \frac{1}{(1 + \Delta)^4}$$

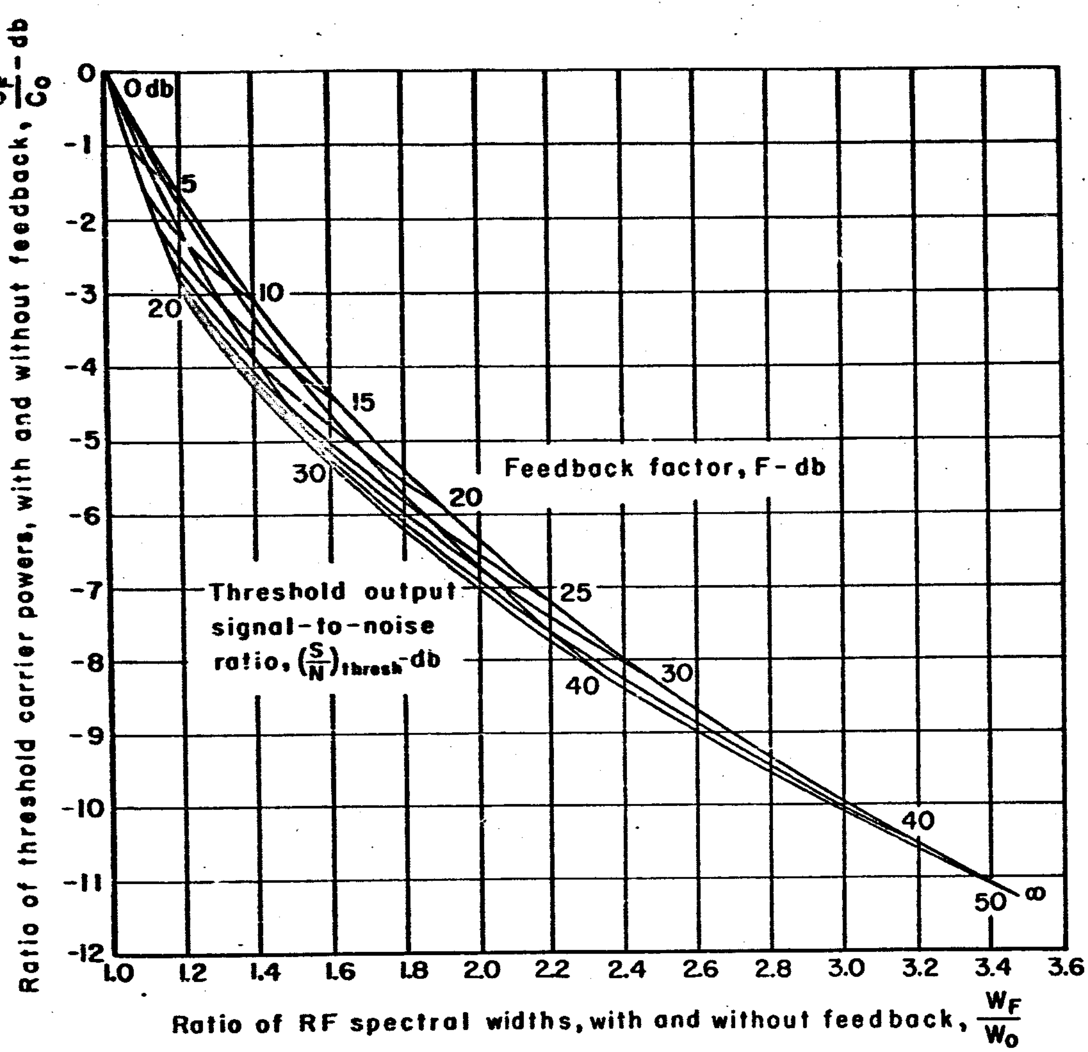

Fig. 6 — Power-bandwidth trade-offs for optimum feedback FM systems

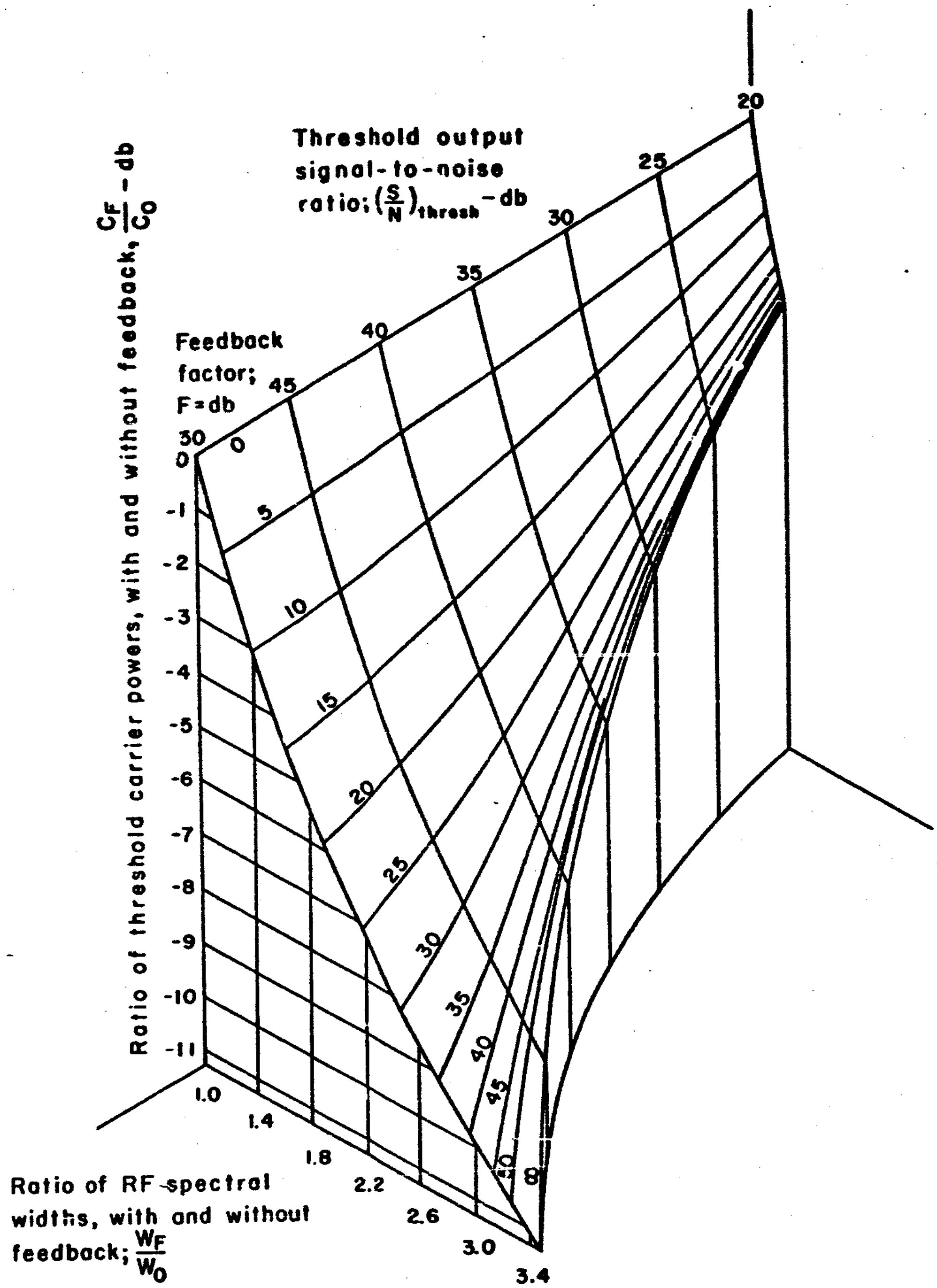

Fig. 7 — Power-bandwidth trade-offs in isometric relief

which, together with Fig. 6, portrays clearly the diminishing returns from the use of large amounts of feedback. For instance, a system having a threshold output signal-to-noise ratio of 40 db has a maximum carrier-power reduction capability of 8.2 db. A feedback factor of 30 db (equivalent to a modulation-index reduction of 31.6) achieves 6.8 db of this amount. Increasing the feedback factor to 40 db (a modulation-index reduction of 100) yields only an additional 0.8 db power saving, while increasing the channel bandwidth from 2.0 to 2.2 times that of the system without feedback.

Figures 6 and 7 show that substantial amounts of carrier power saving are possible only in high-quality systems. However, the modest savings of carrier power possible in low-quality systems require only slight bandwidth increases. For example, a power saving of as much as 10 db is possible only in a system having a 45-db threshold output signal-to-noise ratio and requires, incidentally, tripling the channel bandwidth. On the other hand, a system with a threshold output signal-to-noise ratio of 20 db can achieve a 3-db power saving with only a 20 per cent increase in bandwidth.

V. SYSTEMS COMPARISON

The utility of the power-bandwidth trade-off as an aid in evaluating systems can be enhanced by displaying the trade-off simultaneously for various systems of comparable output performance. An example of interest is that of a single satellite-borne relay serving as the transoceanic element of a telephone system. For a multichannel system, the weakest link will likely be the satellite-to-earth repeater, which will have a limited capability for radiating signal power. Thus, modulation techniques which make the most efficient use of the available power will be preferred, providing the problems of circuit complexity, spectrum utilization and multiplexing are reasonable.

For the circuit cited, frequency modulation (FM), with or without feedback, and pulse-code-modulation (PCM) can be considered as feasible modulation techniques which effect a power-bandwidth trade-off. These will be compared by referencing them to single-sideband (SSB) modulation, since it has a channel bandwidth equal to the modulation baseband, i.e.,

$$W_{SSB} = B \tag{25}$$

and since its output signal-to-noise ratio is equal to the carrier-to-noise* ratio in the channel

$$\left(\frac{S}{N}\right)_{SSB} = \frac{C_{SSB}}{Bn} \tag{26}$$

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*The term "carrier" is used loosely here to denote the total radiated signal power. The SSB system considered is actually a suppressed-carrier system, so the entire radiated signal power appears as sideband energy.

The corresponding relations for FM are given by Eqs. (12) and (13). For equal threshold output signal-to-noise ratios, equating Eqs. (26) and (12) yields the carrier-power ratio

$$\frac{C_{FM}}{C_{SSB}} = \frac{2}{3M^2} \tag{27}$$

The channel-width ratio, from Eqs. (25) and (13) is

$$\frac{W_{FM}}{W_{SSB}} = 2(1 + M) \tag{28}$$

The performance characteristics of PCM have been well investigated.[8-11] It need only be said in review that the intelligence in baseband B cps is sampled every 1/2B sec at the transmitter and that these samples, in a binary N-digit system, are quantized to one of 2^N discrete levels which are then each represented by unique sequences of N binary pulses (i.e., on-off, mark-space, etc.). On reception of the pulses, the N-pulse sequences are reconstructed into the sample values from which the original signal is then reconstituted.

It is apparent that two sources of noise are present. One is the error noise due to faulty reception of the individual binary pulses, and the other is the quantization noise produced by the granularity of the quantizing process. Though different in origin, their effects, when considered as noise interference, are not significantly different and, since they are independent, their mean square values are simply added to determine the total noise.

The quantization noise, of course, depends only on the number of digits used and leads, for a signal occupying each of the 2^N quantization levels with equal probability, to an output signal-to-noise ratio,

$$\left(\frac{S}{N}\right)_{quant} = 4^N - 1 \tag{29}$$

The error noise falls off exponentially with the input carrier-to-noise ratio, since it depends solely on the pulse error probability, p,

$$\left(\frac{S}{N}\right)_{error} = \frac{1}{4pq} - 1, \quad p + q = 1 \tag{30}$$

A well-defined threshold occurs at the point where the signal-to-noise ratios due to quantization and pulse errors are equal. When the signal level is only 1 db above this threshold, the output signal-to-noise ratio is determined almost exclusively by Eq. (29). The error probability at the threshold is

$$p(1 - p) = 4^{-(N + 1)} \tag{31}$$

Reiger[12] shows that the error probability for an optimum linear detection system is related to the normalized signal energy, $\alpha = E/n$, by

$$p = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\alpha}}^{\infty} \exp(-y^2)dy \tag{32}$$

where E is the total pulse energy. Since the pulses arrive at the rate of 2NB/sec, the carrier power of the received PCM pulse train then becomes

$$C_{PCM} = 2NBE = 2NBn\alpha \tag{33}$$

The carrier power required for a SSB system having the same output signal-to-noise ratio as a PCM system is found by equating Eqs. (26) and (29), yielding

$$\frac{C_{SSB}}{Bn} = 4^N - 1 \tag{34}$$

The ratio of carrier powers then becomes

$$\frac{C_{PCM}}{C_{SSB}} = \frac{2N\alpha}{4^N - 1} \tag{35}$$

The normalized signal energies and output signal-to-noise ratios at threshold for 6, 7 and 8-digit PCM systems are listed below:

N	α	(S/N)
6	8.7 db	36.1 db
7	9.4 db	42.1 db
8	10.0 db	48.2 db

The normalized signal energies can be reduced by 1.4 db if weighted PCM[11] is used; noncoherent detection imposes a 4-db penalty.

As for spectral occupancy, the PCM pulse train of 2NB pulses/sec can, in theory, occupy a bandwidth as little as NB cps. Practical systems, however, will usually occupy more spectrum, so this fact will be expressed in a PCM channel-width formula

$$W_{PCM} = kNB , \quad k \geq 1 \tag{36}$$

by the introduction of a shape factor k. Present techniques indicate that a practical limit for k in a binary system is somewhere between 2 and 3. The ratio of bandwidths, from Eqs. (25) and (36) is

$$\frac{W_{PCM}}{W_{SSB}} = k\, N \tag{37}$$

Equations (27), (28), (35) and (37) are plotted as normalized trade-offs in Figs. 8, 9 and 10 for 6, 7, and 8-digit PCM systems, respectively, each with a comparable FM system. The shape factor k and the feedback factor F are shown as parameters. The values of modulation index M to be used in Eqs. (27) and (28) are obtained from Fig. 5.

Apart from considerations of circuit and system complexity, the trade-offs indicate that an optimum PCM system is superior to a conventional FM system by from 2.5 to 3.5 db, for the systems shown, in its utilization of transmitted power. In theory, PCM is also more efficient in its use of radio spectrum but practical factors tend to make good PCM systems ($2 \geq k \geq 3$) occupy approximately the same radio spectrum as conventional FM.

The application of feedback to FM in substantial amounts virtually reverses the situation. The FM systems shown then require 4 to 6 db less power than comparable PCM systems, but the transmitted spectrum is increased considerably. For example, the 42-db feedback-FM system requires about twice the spectrum of a comparable 7-digit PCM system of good design. FM systems of higher quality are penalized severely in the bandwidth-power trade-off as compared with PCM systems.

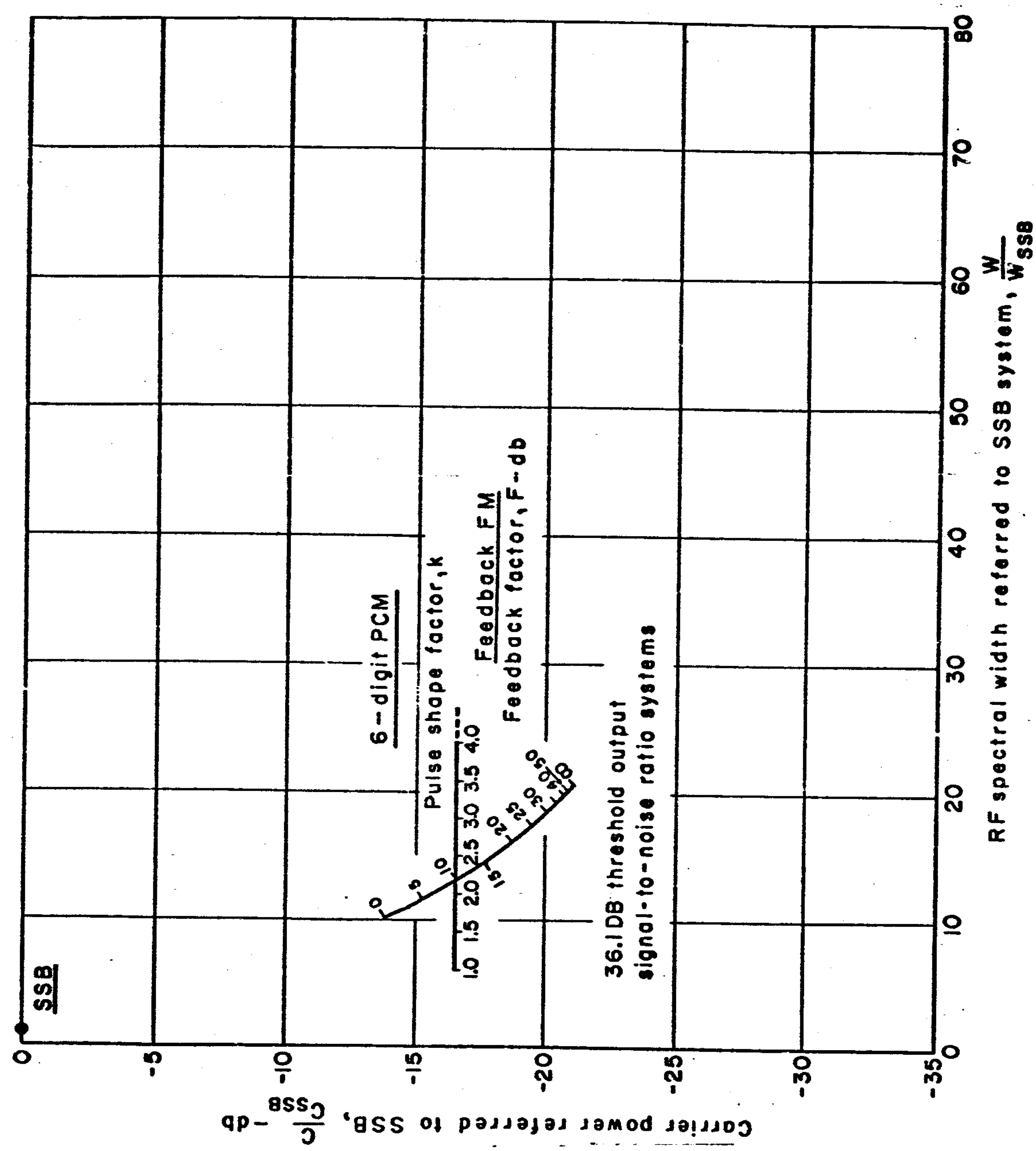

Fig. 8 — Comparison of 36-db feedback FM and 6-digit PCM systems

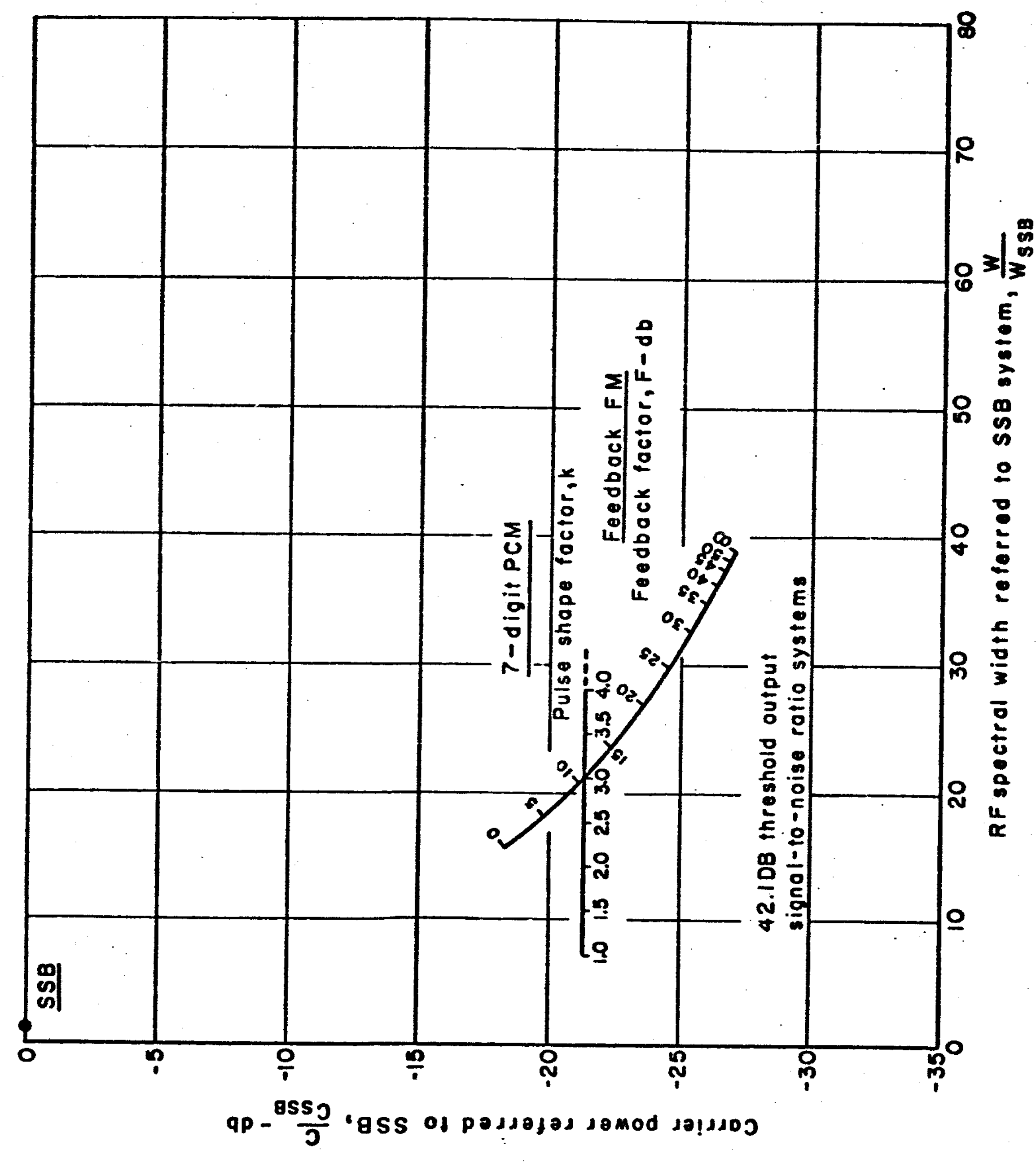

Fig. 9 —— Comparison of 42-db feedback FM and 7-digit PCM systems

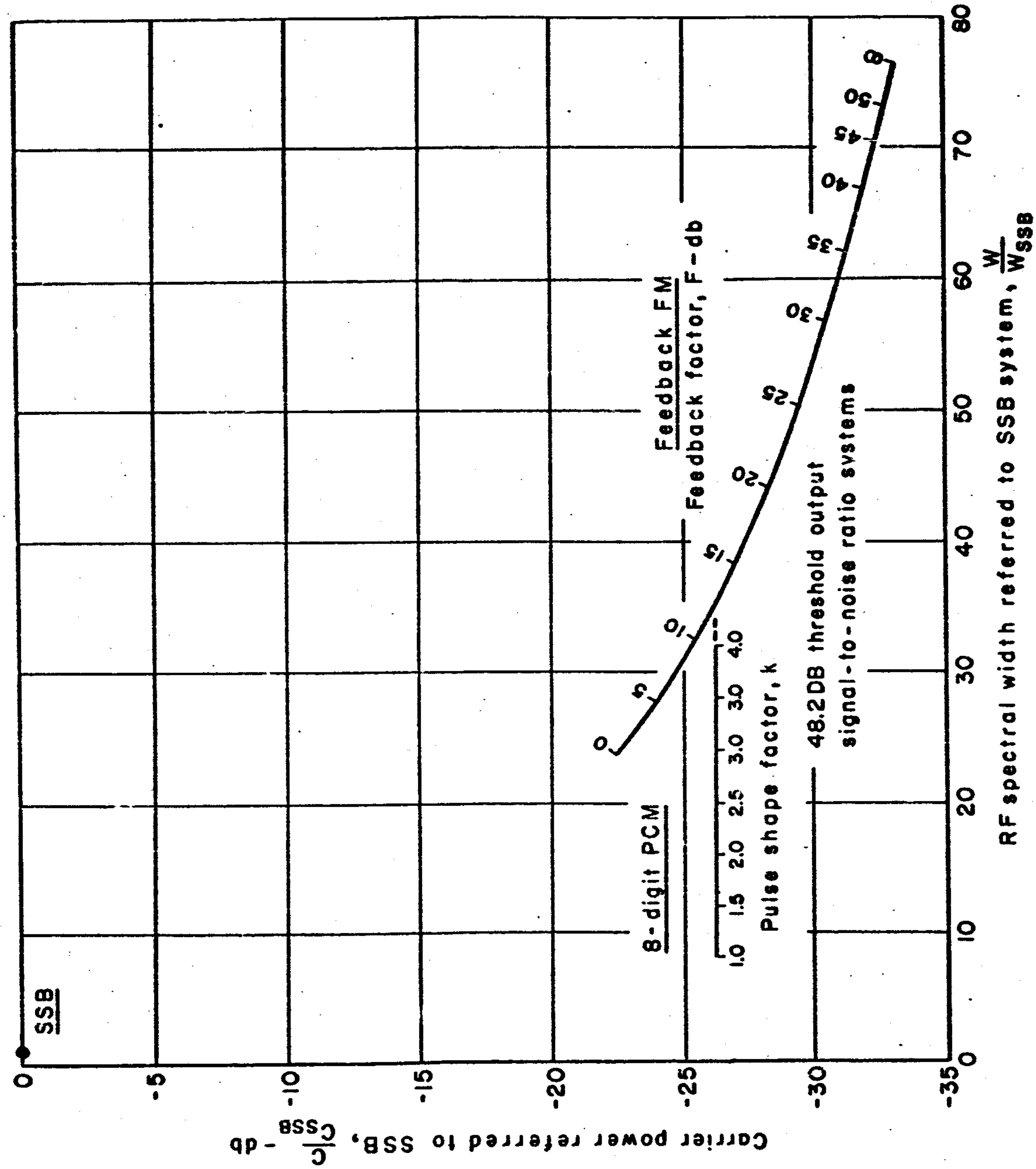

Fig.10 —— Comparison of 48-db feedback FM and 8-digit PCM systems

VI. CONCLUSION

The analysis indicates that the use of feedback in an FM communication system will permit a power-bandwidth trade-off which can result in power savings of as much as 8 db, depending on the threshold signal-to-noise ratio desired. The radio spectrum required for the transmission will be increased by a factor of two or more compared with a conventional FM system of the same quality.

It is important to optimize the design of FM systems by using the modulation index which permits the system to attain the minimum acceptable output signal-to-noise ratio at its improvement threshold. Non-optimum modulation indexes, particularly lower ones, can increase the power requirement for a given grade of service by as much as 10 db or more.

The degree of the power-bandwidth trade-off achieved depends on the amount of feedback used. For a practical system, much of the indicated improvement can be attained by the use of relatively small amounts of feedback, i.e., 15 to 20 db. Even if the threshold were to behave in the idealized manner assumed, there would be little point in using more than 40 db of feedback.

Assuming optimum detection, a 7-digit PCM system requires about 3 db less power than a comparable conventional FM system, but about 5 db more power than a feedback FM system using 35 to 40 db of feedback. However, the power-bandwidth trade-off is more favorable for the PCM system, which requires a channel only 14 to 21 times the width of the baseband compared with a value about twice that for the feedback FM system.

While the computations presented here form an important basis for comparison of PCM and feedback FM systems, the choice in any specific application will also be influenced by considerations of circuit complexity, multiplexing problems, etc.

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